Variation of our cutoff criteria for events in which the Λ decays too near to or too far from the Ξ^0 production vertex produces a shift in τ_{Ξ^0} of less than 0.15×10^{-10} sec.

When these systematic uncertainties are included with the statistical and measurement uncertainties, we obtain our final result:

$$\tau_{\Xi^0} = 2.5_{-0.3}^{+0.4} \times 10^{-10} \text{ sec}.$$

DISCUSSION OF RESULTS

The determinations of the Ξ^- and Ξ^0 lifetimes reported to date are summarized with our results in Tables I and II.[19] It is seen that there are no statistically significant discrepancies among these results.

The ratio of the Ξ^0 decay rate to the Ξ^- decay rate provides a sensitive test to the rule $|\Delta T| = \frac{1}{2}$ rule for nonleptonic, strangeness-changing weak decays.[12] Our result, $\lambda_{\Xi^0}/\lambda_{\Xi^-} = 0.68 \pm 0.10$, is within two standard deviations of the $|\Delta T| = \frac{1}{2}$ prediction of 0.5.

ACKNOWLEDGMENTS

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[19] The weighted averages of the Ξ^- and Ξ^0 lifetimes quoted in Tables I and II are $\tau_{\Xi^-} = 1.75 \pm 0.05 \times 10^{-10}$ sec and $\tau_{\Xi^0} = 3.1_{-0.3}^{+0.4} \times 10^{-10}$ sec. The resulting ratio of decay rates, $\lambda_{\Xi^0}/\lambda_{\Xi^-} = 0.57 \pm 0.07$, is within one standard deviation of the $|\Delta T| = \frac{1}{2}$ prediction. Because of the different methods used in the lifetime analyses by the various groups, such "world averages," although interesting, may not be significant.

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Weak Corrections to the Magnetic Moments of Leptons*

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A calculation of the contribution of the weak interactions to the magnetic moments of the leptons is reported. An intermediate vector boson is assumed, and lowest order effects are calculated. The radiative corrections to these results are estimated by making use of the summation technique introduced by Lee. The contributions are small compared to the uncertainties in the experimental values of the magnetic moments. For the muon the changes in the magnetic moment are at least an order of magnitude smaller than the contributions of the strong interactions. If the muon's neutrino has a relatively large mass, its induced magnetic moment will affect the neutrino scattering cross sections calculated by Bernstein and Lee.

I. INTRODUCTION

THE electromagnetic properties of the leptons (and the intermediate vector boson W) are of particular interest: For all of the other particles the significance of at least some of these properties is complicated by the presence of strong interactions. The values of the charge and the magnetic moment of the electron and the associated errors have been given by Du Mond.[1] The corresponding quantities for the muon have been given by Shapiro and Lederman.[2] Recently, Bernstein, Ruderman, and Feinberg[3] have indicated the uncertainties in the experimental values of the charges, magnetic moments, and charge radii of the neutrinos.

In this paper we calculate the contributions of the weak interactions to the magnetic moments of the leptons. We assume that the weak interactions are mediated by a vector boson. The lowest order contributions to the moments of the charged leptons are calculated in Sec. II. In Sec. III a similar analysis is carried out for the neutrinos. Section IV consists of an an estimate of the effect of higher order electromagnetic and weak interactions upon the leptonic magnetic moments. The conclusions follow in Sec. V.

II. CHARGED LEPTONS

We note, to begin with, that the electromagnetic properties of the W particle have been the subject of some discussion.[4] The suggestion is that the assumption

* Supported by the U. S. National Science Foundation.
[1] J. W. M. DuMond, Ann. Phys. (Paris) **7**, 365 (1959).
[2] G. Shapiro and L. M. Lederman, Phys. Rev. **125**, 1022 (1962).
[3] J. Bernstein, M. Ruderman, and G. Feinberg, Phys. Rev. **132**, 1227 (1963).
[4] J. Bernstein and T. D. Lee, Phys. Rev. Letters **11**, 512 (1963); Ph. Meyer and G. Salzman, Nuovo Cimento **14**, 1310 (1959); R. A. Shaffer, Phys. Rev. **131**, 2203 (1963).

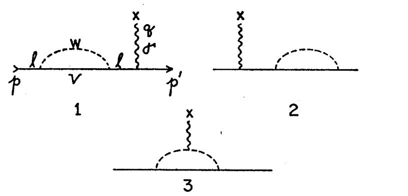

FIG. 1. Feynman diagrams which represent the lowest-order weak interaction contributions to the electromagnetic properties of the charged leptons.

of a minimal electromagnetic interaction for this particle is consistent with the electromagnetic properties of the well-known particles. In addition, the minimal vector boson electrodynamics are the least pathological from the standpoint of ultraviolet divergences.

We assume, then, the interaction Lagrangian given by[5]

$$L_1 = -ie(A_\mu \phi_\nu^* \partial^\mu \phi^\nu - A_\mu \phi_\nu^* \partial^\nu \phi^\mu - \partial^\mu \phi_\nu^* A_\mu \phi^\nu + \partial_\nu \phi_\mu^* A^\mu \phi^\nu) - e^2(A_\mu A^\mu \phi_\nu^* \phi^\nu - A^\mu A^\nu \phi_\mu^* \phi_\nu), \quad (1)$$

where ϕ^μ creates a W^+. The interaction of leptons with the W particle is given by

$$L_2 = ig\bar{\Psi}_l \gamma_\mu (1+i\gamma_5) \Psi_\nu \phi^\mu + \text{H.c.}, \quad (2)$$

where g is a semiweak coupling constant, l is either μ or e, and ν refers to the appropriate neutrino. We also assume the usual electromagnetic interaction for the charged leptons.

We now consider the lowest order weak interaction contributions to the electromagnetic properties of the charged leptons. The appropriate Feynman diagrams are shown in Fig. 1. Counting powers we find that the corresponding expressions are highly divergent. However, if we wish only the contribution to the magnetic moment of "l" we must extract the coefficient of the term $\bar{\Psi}_l \sigma_{\mu\nu} q^\mu A^\nu \Psi_l$, where q^μ is the momentum transferred to the electromagnetic field. The only contribution is from the expression corresponding to diagram 3, and it is logarithmically divergent.[6] If we insert a cutoff into the neutrino propagator we obtain[7]

$$M_{3\Gamma} \approx g^2 e M^2 / [2^4 \pi^{5/2} (EE'q_0)^{1/2} m^2] \times [\ln(\Lambda^2/m^2) - 1] \bar{\Psi}_l \sigma_{\mu\nu} q^\mu A^\nu \Psi_l, \quad (3)$$

where M is the charged lepton's mass, m the vector boson's mass, E and E' the initial and final lepton energies, respectively, and Λ the cutoff.

This expression yields a change in the magnetic moment of l equal to

$$\Delta\Gamma \simeq -g^2 M^2 / (4\pi^2 m^2) [ln(\Lambda^2/m^2) - 1], \quad \text{lepton magnetons.} \quad (4)$$

We have[8]

$$g^2/m^2 = G/\sqrt{2},$$

and

$$G \simeq 1.0 \times 10^{-5} / M_p^2,$$

where M_p is the proton's mass.

Therefore, for the muon[9]

$$\Delta\Gamma_\mu \simeq -2 \times 10^{-9} [\ln(\Lambda^2/m^2) - 1], \quad \text{muon magnetons.} \quad (5)$$

For a wide range of Λ this contribution is very small compared to the uncertainty in the experimental value of Γ_μ which is about 5×10^{-6}.[10] More significantly, it is much smaller than the contributions of the strong interactions to Γ_μ. These contributions have been estimated to be about 1×10^{-7}.[11]

We note that an intrinsic vector boson magnetic moment of one boson magneton will produce a logarithmic divergence which just cancels the divergence shown above.[12] However, the difficulties introduced by such an interaction have already been mentioned.

III. NEUTRINOS

If neutrinos have zero mass and are described by two-component spinors their electromagnetic properties

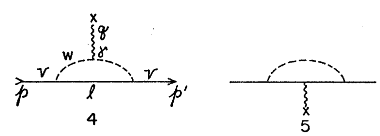

FIG. 2. Feynman diagrams representing weak interaction contributions to the electromagnetic properties of the neutrinos.

are determined by a single form factor[13]:

$$\langle \nu' | J_\mu | \nu \rangle = i\bar{u}' \gamma_\mu (1+i\gamma_5) u \cdot F(q^2), \quad (6)$$

with $F(0) = 0$.

However, if we assume that neutrinos have finite mass, they may have a magnetic moment induced by the expressions corresponding to the diagrams of Fig. 2. Again, the coefficient of the magnetic moment term is logarithmically divergent. Since both virtual particles

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[5] See, e.g., R. A. Shaffer, Phys. Rev. **128**, 1452 (1962).

[6] We do not consider the leptons to have intrinsic magnetic moment interactions. Therefore, we may not call this divergent expression a renormalization constant.

[7] We cut off the neutral particle's propagator in order to maintain gauge invariance. See, e.g., R. A. Carhart, Phys. Rev. **132**, 2337 (1963). We assume here that $M_\nu = 0$ and that q^2 and $M^2 \ll m^2 \ll \Lambda^2$.

[8] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

[9] After the present work was completed several other calculations of $\Delta\Gamma_\mu$ appeared in the literature: R. D. Amado and L. Holloway, Nuovo Cimento **30**, 1083 (1963), obtained $\Delta\Gamma_\mu \simeq 2.0 \times 10^{-7}$. [See, however, R. D. Amado and L. Holloway, Nuovo Cimento **30**, 1572 (1963).] G. Segrè, Phys. Letters **7**, 357 (1963), obtained $\Delta\Gamma_\mu \simeq -1.0 \times 10^{-9} [1 - 3\ln(\Lambda^2/m^2)]$. Ph. Meyer and D. Schiff, Phys. Letters **8**, 217 (1964), obtained our result.

Segrè used the ξ-limiting process of Lee and Yang. As Meyer and Schiff point out, this method of introducing a cutoff should yield the same result for the $\ln(\Lambda/m)$ term in $\Delta\Gamma_\mu$ as we obtained but may give a different result for the finite term. We have calculated $\Delta\Gamma_\mu$ using the ξ-limiting method and obtain $\Delta\Gamma_\mu \simeq -2.0 \times 10^{-9} \times [\ln(\Lambda^2/m^2) - 5/3]$.

[10] G. Charpak, F. J. M. Farley, R. L. Garwin, T. Muller, J. C. Sens, and A. Zichichi, Phys. Letters **1**, 16 (1962).

[11] L. Durand, III, Phys. Rev. **128**, 441 (1962).

[12] See, e.g., M. E. Ebel and F. J. Ernst, Nuovo Cimento **15**, 173 (1960), and the second paper of Ref. 4.

[13] See the first paper of Ref. 4 and the last paper of Ref. 9.

are charged there is an ambiguity in the method of introducing a cutoff. What is significant, however, is that the magnetic moment's coefficient depends upon $(M_\nu/m)^2$. With the present upper limit on the muon neutrino's mass the magnetic moment is only on the order of 10^{-12} Bohr magnetons. Unfortunately, the present experimental upper limit on Γ_{ν_μ} is about 10^{-8} Bohr magnetons.[3]

IV. HIGHER ORDER EFFECTS

The electromagnetic and weak interactions of vector bosons are unrenormalizable. Lee and Yang, and Lee[14] have developed a method of handling the leading divergences in the perturbation expansion of vector boson electrodynamics. The general effect of the technique is to replace the term $\ln(\Lambda/m)$ with a term $-\frac{1}{2}\ln\alpha$.[15]

The leading divergences encountered in a certain class of expressions involving semiweak vector boson interactions have been summed by Feinberg and Pais.[16] The general effect of this summation procedure is to redefine the coupling constant g and to replace m^2 with $3/4m^2$. Applying the above considerations to the computation of the leptonic magnetic moments we see that $\Delta\Gamma_\mu$ may be increased to about 10^{-8} muon magnetons, and that the upper limit on $\Delta\Gamma_{\nu_\mu}$ might be raised to about 10^{-11} Bohr magnetons.

V. CONCLUSIONS

The above results suggest that the effects of weak interactions upon the magnetic moments of the leptons would be difficult if not impossible to observe. In the case of the muon (and the electron) the strong interactions are of more significance than the weak interactions in altering the magnetic moments. For the electronic neutrino the small mass value permits only a negligible magnetic moment.

If the muon's neutrino is shown to have a relatively large mass, and the sensitivity of neutrino scattering experiments is increased significantly, the observation of magnetic moments induced by weak interactions might be possible. Regardless of the neutrino's mass, scattering experiments involving large momentum-transfers will be dominated by the form factor calculated by Bernstein and Lee.

ACKNOWLEDGMENTS

The author would like to thank Professor W. G. Holladay for helpful discussions during the course of this work.

[14] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962); and T. D. Lee, Phys. Rev. **128**, 899 (1962).
[15] See Ref. 13.
[16] G. Feinberg and A. Pais, Phys. Rev. **131**, 2724 (1963); Y. Pwu and T. T. Wu, Phys. Rev. **133**, B778 (1964); see also the remarks at the end of the paper by Bernstein and Lee.

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Kaon Decays and Pion Statistics*

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Selection rules for nonleptonic decays of kaons are given under different assumptions about the statistics of pions. It is found that the observed decay modes are compatible only with the assumption that pions are bosons.

Several authors[1–3] have recently emphasized that the statistics of elementary particles should be determined directly without appeal to the spin-statistics theorem. This is particularly important since the indistinguishability of identical particles does not by itself imply that particles can obey only Bose-Einstein or Fermi-Dirac statistics.[4] The possiblity that particles obey generalized statistics exists *a priori*. The purpose of this note is to present arguments that pions obey Bose-Einstein statistics.

Our arguments are based on selection rules for the nonleptonic decays of kaons. We make explicit use of the following assumptions:

I. Conservation of angular momentum.

II. *CP* conservation. This conservation law is the most probable mechanism inhibiting the two pion decay of K_2^0.[5]

III. The kaon spin is zero. Evidence for this assignment independent of the statistics of pions includes the

* Supported in part by the National Science Foundation.
[1] O. W. Greenberg and A. Messiah (private communication to W. G. Holladay).
[2] D. B. Lichtenberg, in *Proceedings of Athens Conference on Resonant Particles*, edited by B. A. Munir and L. J. Gallaher (Ohio University Press, Athens, Ohio, 1963), p. 152.
[3] R. Gatto, Phys. Letters **5**, 56 (1963).
[4] A. Galindo, A. Morales, and R. Nuñez-Lagos, J. Math. Phys. **3**, 324 (1962).

[5] R. G. Sachs, Ann. Phys. (N. Y.) **22**, 239 (1963).